Uitwerkingen

1a. Normeren: $A\int_0^\infty xe^{-x^2}dx = [-\frac{A}{2}e^{-x^2}]_0^\infty = \frac{A}{2} = 1$ zodat $A = 2$

b. $\langle x\rangle = 2\int_0^\infty x^2e^{-x^2}dx = \frac{1}{2}\sqrt{\pi}$

c. $\sigma^2 = \langle x^2\rangle - \langle x\rangle^2 = 2\int_0^\infty x^3e^{-x^2}dx - (\frac{1}{2}\sqrt{\pi})^2 = 1 - \frac{1}{4}\pi \rightarrow \sigma = \sqrt{1-\frac{1}{4}\pi} \approx 0{,}46$

2a. De omtrek is: $O = a + c + \sqrt{c^2 - a^2} = 14{,}4 + 22{,}4 + \sqrt{22{,}4^2 - 14{,}4^2} = 54{,}0\,cm$

$$\sigma_O^2 = (\frac{\partial O}{\partial a}\Delta a)^2 + (\frac{\partial O}{\partial c}\Delta c)^2 = [(1 + \frac{a}{\sqrt{c^2-a^2}})\Delta a]^2 + [(1+\frac{c}{\sqrt{c^2-a^2}})\Delta c]^2 = 0{,}614$$

dus $\sigma_O = 0{,}8\,cm$

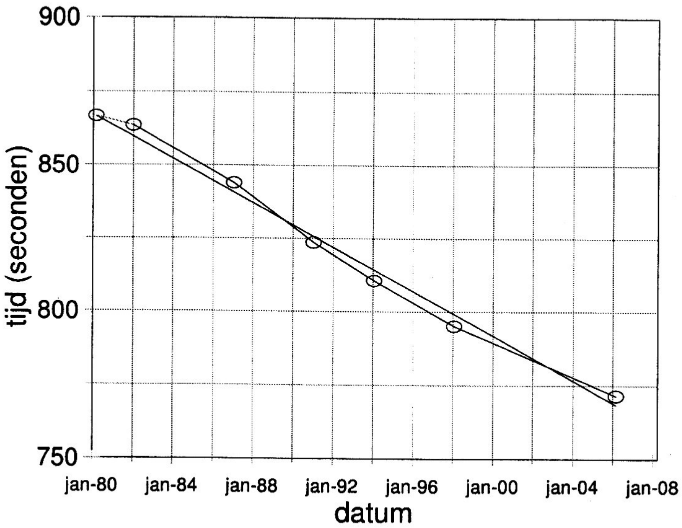
De omtrek is derhalve: $O = 54{,}0 \pm 0{,}8\,cm$

b. Het gewogen gemiddelde is: $\langle O\rangle = \dfrac{O_1.\frac{1}{\sigma_1^2} + O_2.\frac{1}{\sigma_2^2}}{\frac{1}{\sigma_1^2}+\frac{1}{\sigma_2^2}} = 55{,}07$

Voor de fout in het gemiddelde geldt: $\frac{1}{\sigma_{\langle O\rangle}^2} = \frac{1}{\sigma_1^2} + \frac{1}{\sigma_2^2} = 6{,}04$ dus $\sigma_{\langle O\rangle} = 0{,}4$

Het gewogen gemiddelde is dus: $\langle O\rangle = 55{,}1 \pm 0{,}4\,cm$

3a.



b. De som van de kwadraten is:

$$M(a,b) = \sum_1^4 (T_i - a.D_i - b)^2$$

Uit $\frac{\partial M}{\partial a} = 0$ en $\frac{\partial M}{\partial b} = 0$ volgt:

$$a = \frac{\langle xy\rangle - \langle x\rangle\langle y\rangle}{\langle x^2\rangle - \langle x\rangle^2} = \frac{3228677 - 4036{,}143*824{,}9914}{25853879 - 4036{,}143^2} = -0{,}01057\,s/dag = -3{,}859\,s/jaar$$

c. De verwachte tijd is: $866{,}71 - \frac{3{,}859}{365}.9815 = 762{,}94$ seconden,

de gerealiseerde tijd van Sven Kramer is: 769,88 seconden; dus een verschil van bijna 7 s.